

Problem Set #3

Exercise 1: (★) 3 points

Show that if n is a positive integer, then $\mu(n)\mu(n+1)\mu(n+2)\mu(n+3) = 0$.

Exercise 2: (★) 3 points

Define the arithmetic function Λ by:

$$\Lambda(n) = \begin{cases} \log(p) & n = p^k \\ 0 & \text{otherwise} \end{cases}$$

Show that $\sum_{d|n} \Lambda(d) = \log(n)$ whenever n is a positive integer.

Exercise 3: (★) 3 points

Let $g(n)$ be the arithmetic function whose summary function is $f(n) = \sum_{d|n} g(d) = \frac{n-1}{n+1}$. Find $g(24)$.

Exercise 4: (★) 6 points

Let $g(n)$ be the arithmetic function whose summary function is $f(n) = \sum_{d|n} g(d) = \phi(n)/n$.

1. Prove that $g(n)$ is multiplicative.
2. Calculate $g(p^\alpha)$ where p is a prime and $\alpha \geq 1$.
3. Calculate $g(n)$ (for $n \geq 2$, your answer should depend on the prime factorization of n).

¹(★) = easy, (★★) = medium, (★★★) = challenge